QUANTUM STATES DISTINGUISHABILITY USING LOCC

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OUTLINE

- Introduction – What is distinguishability?
- Why distinguishability?
- Locally – one party
- Locally – many parties – w/o communication
- Multi-partite with communication
- Unextendible product bases (UPBs)
- Distinguishability of a UPB

WHAT IS DISTINGUISHABILITY?

CLASSICALLY

We can (at least in principle), distinguish between the different states of an entity (e.g. a macroscopic object)

Example: A die has six distinguishable states.

WHAT IS DISTINGUISHABILITY?

QUANTUMLY

Given a state $|\phi_i\rangle$ from a set of known states $\{|\phi_i\rangle\}$, can we find a protocol that will identify with probability 100% which of the states it is? (i.e. its index *i*)

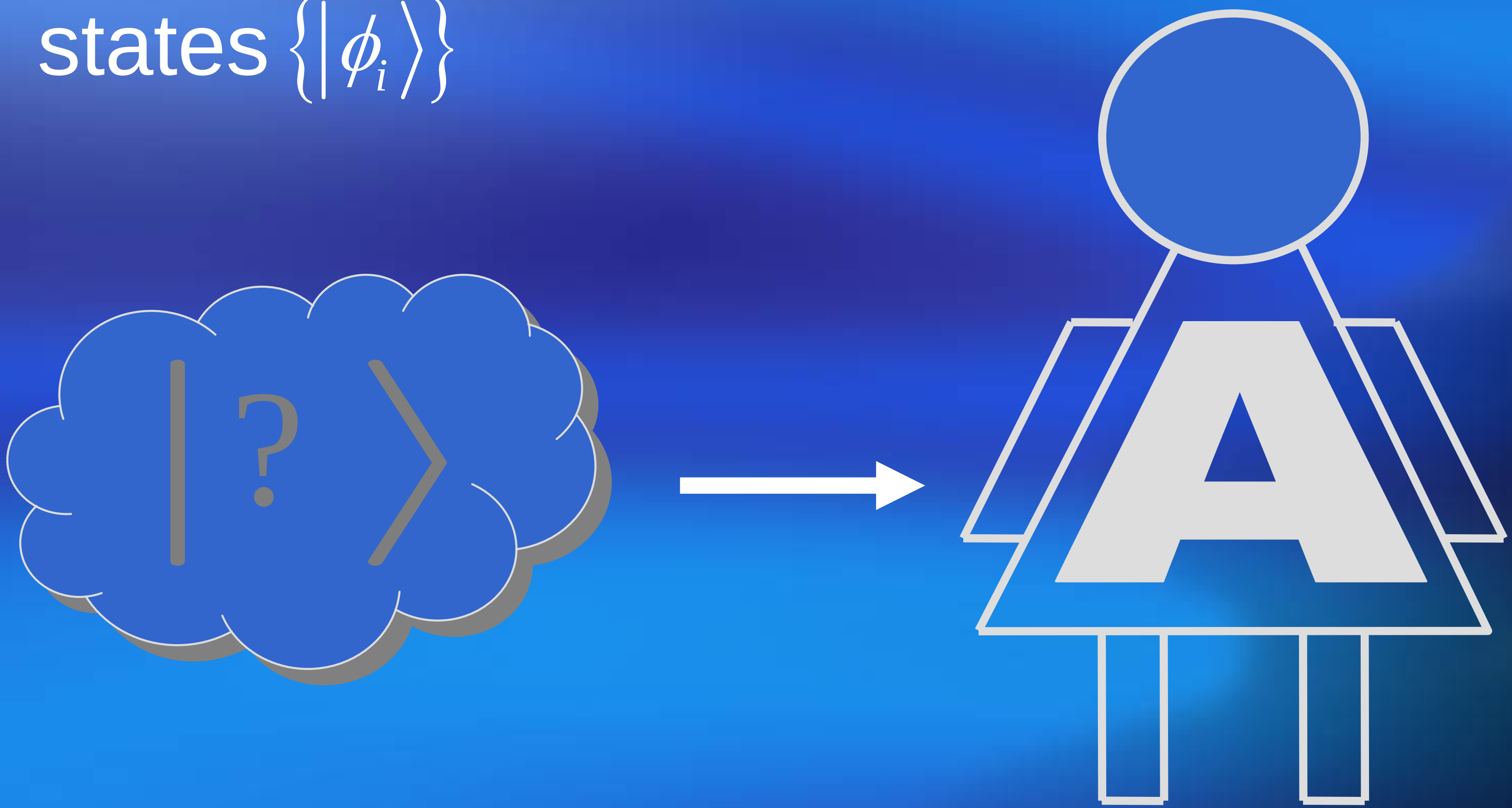
WHY DISTINGUISHABILITY?

- Distinguish different physical states → extract information
- Data hiding
- Metaphysical implications (locality/causality)
- Difference between classical and quantum communication
- Test our knowledge/comprehension of QM

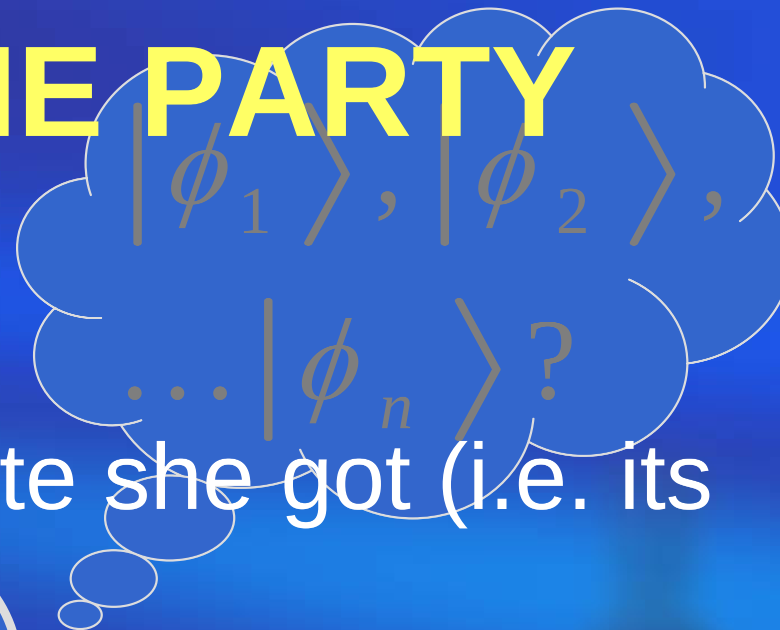
LOCALLY – ONE PARTY

SETTING

Alice is given a state $|\phi_i\rangle$ from a known set of states $\{|\phi_i\rangle\}$



LOCALLY – ONE PARTY



PROBLEM

Can Alice identify which state she got (i.e. its index *i*)?

LOCALLY – ONE PARTY

PROBLEM

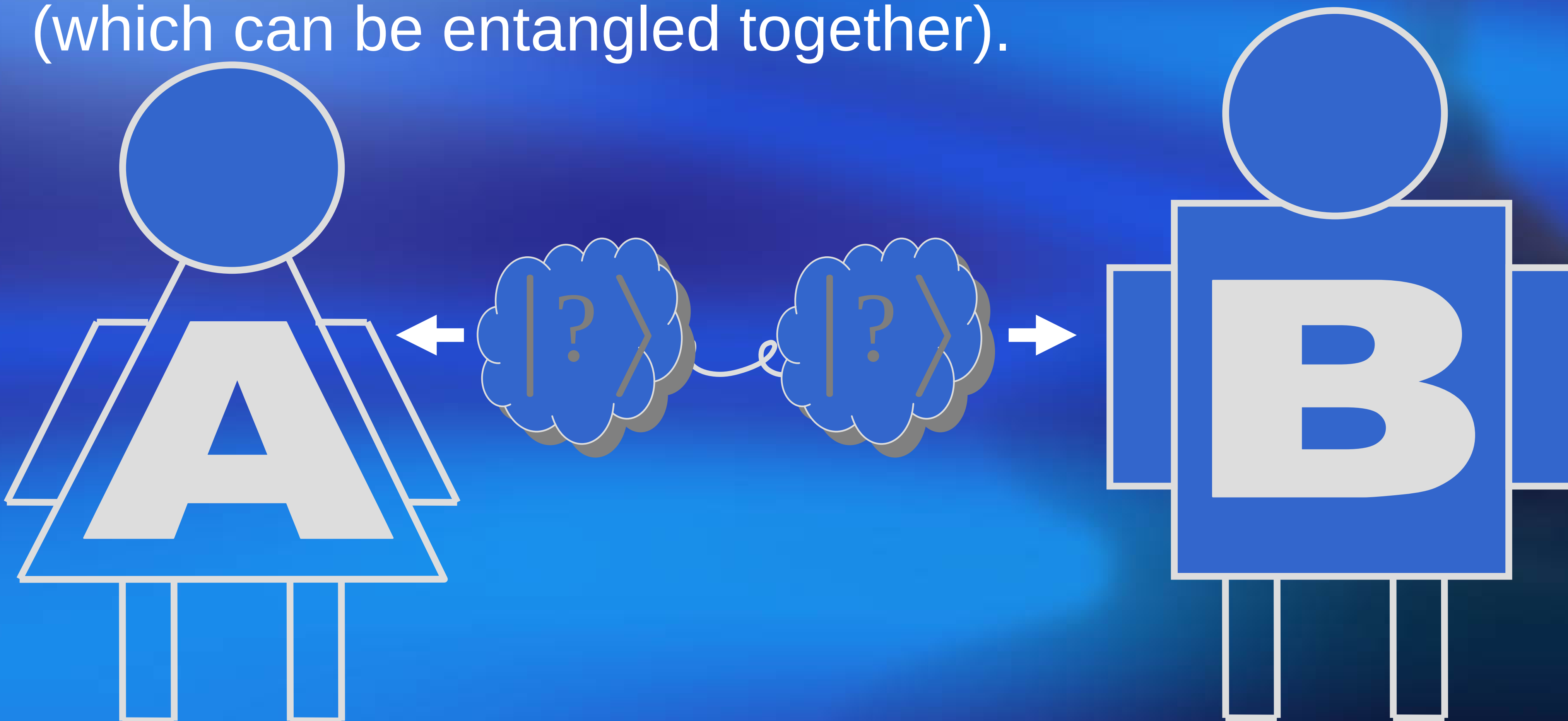
Given a state $|\phi_i\rangle$ from a set of known states $\{|\phi_i\rangle\}$, can we identify which one it is? (i.e. its index *i*)

YES, iff the $|\phi_i\rangle$s are orthogonal *(even if we consider generalized measurements i.e. POVMs)*

MULTI-PARTITE – W/O COMM.

SETTING

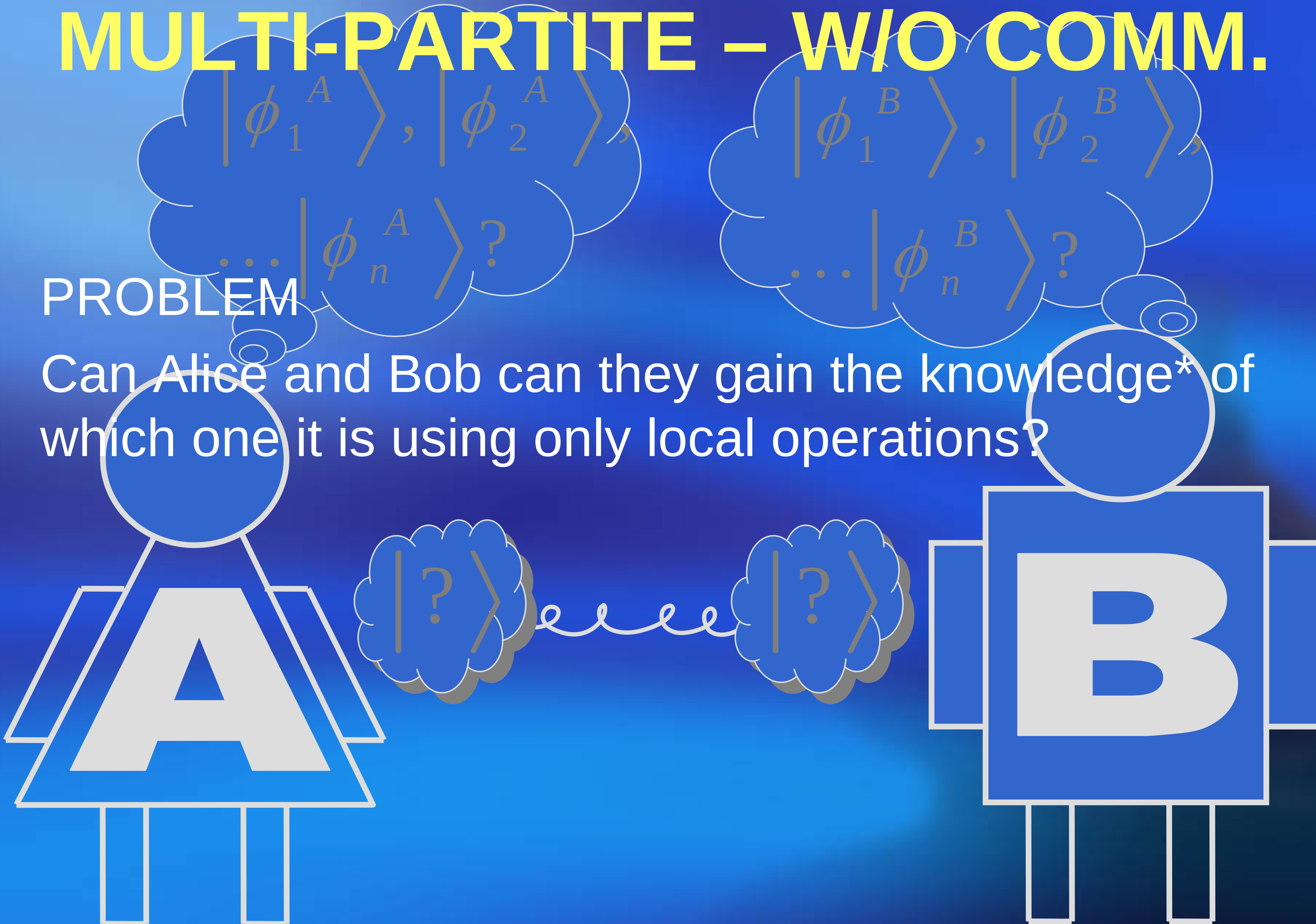
A state $\left|\phi_i\right\rangle$ from a set $\left\{\left|\phi_i\right\rangle\right\}$ is seperated into two parts. Alice and Bob are each given one part (which can be entangled together).



MULTI-PARTITE – W/O COMM.

PROBLEM

Can Alice and Bob can they gain the knowledge* of which one it is using only local operations?



MULTI-PARTITE – W/O COMM.

RESTRICTIONS

- The two parties have only one copy of their part of the state
- They can only perform local operations
- They cannot communicate

EXAMPLE 1

$$\{|\phi_i\rangle\} = \begin{cases} |\phi_1\rangle = |0\rangle_A |0\rangle_B \\ |\phi_2\rangle = |0\rangle_A |1\rangle_B \\ |\phi_3\rangle = |1\rangle_A |0\rangle_B \\ |\phi_4\rangle = |1\rangle_A |1\rangle_B \end{cases}$$

EXAMPLE 1

$$\left|\phi_1\right\rangle = \left|0\right\rangle_A \left|0\right\rangle_B$$

$$\left|\phi_2\right\rangle = \left|0\right\rangle_A \left|1\right\rangle_B$$

$$\left|\phi_3\right\rangle = \left|1\right\rangle_A \left|0\right\rangle_B$$

$$\left|\phi_4\right\rangle = \left|1\right\rangle_A \left|1\right\rangle_B$$

Alice measures in the $\{\left|0\right\rangle, \left|1\right\rangle\}$ basis and gets a 1, she now knows what was the state of the first qubit.

EXAMPLE 1

$$|\phi_1\rangle = |0\rangle_A |0\rangle_B$$

$$|\phi_2\rangle = |0\rangle_A |1\rangle_B$$

$$|\phi_3\rangle = |1\rangle_A |0\rangle_B$$

$$|\phi_4\rangle = |1\rangle_A |1\rangle_B$$

Bob measures in the same basis and gets a 0, so he knows what was the state of the second qubit. Together they know the state.

EXAMPLE 2

$$\{|\phi_i\rangle\} = \begin{cases} |\phi_1\rangle = |0\rangle_A |0\rangle_B \\ |\phi_2\rangle = |1\rangle_A |+\rangle_B \\ |\phi_3\rangle = |0\rangle_A |1\rangle_B \\ |\phi_4\rangle = |1\rangle_A |-\rangle_B \end{cases}$$

EXAMPLE 2.1

$$\left|\phi_1\right\rangle = \left|0\right\rangle_A \left|0\right\rangle_B$$

$$\left|\phi_2\right\rangle = \left|1\right\rangle_A \left|+\right\rangle_B$$

$$\left|\phi_3\right\rangle = \left|0\right\rangle_A \left|1\right\rangle_B$$

$$\left|\phi_4\right\rangle = \left|1\right\rangle_A \left|-\right\rangle_B$$

Alice measures in the $\{\left|0\right\rangle, \left|1\right\rangle\}$ basis and gets a 1, the states 1 and 3 are eliminate as known by Alice.

EXAMPLE 2.1

$$|\phi_1\rangle = \cancel{|0\rangle_A}|0\rangle_B$$

$$|\phi_2\rangle = |1\rangle_A|+\rangle_B$$

$$|\phi_3\rangle = \cancel{|0\rangle_A}|1\rangle_B$$

$$|\phi_4\rangle = |1\rangle_A|-\rangle_B$$

Now, Bob doesn't know if he should measure in the $\{|0\rangle,|1\rangle\}$ or $\{|+\rangle,|-\rangle\}$ basis. Even if he picks the right one, he still doesn't know if the result is right.

EXAMPLE 2.1

$$|\phi_1\rangle = \xcancel{|0\rangle_A}|0\rangle_B = \xcancel{|0\rangle_A}\left(|+\rangle_B + |-\rangle_B\right)$$

$$|\phi_2\rangle = |1\rangle_A|+\rangle_B = |1\rangle_A\left(|0\rangle_B + |1\rangle_B\right)$$

$$|\phi_3\rangle = \xcancel{|0\rangle_A}|1\rangle_B = \xcancel{|0\rangle_A}\left(|+\rangle_B - |-\rangle_B\right)$$

$$|\phi_4\rangle = |1\rangle_A|-\rangle_B = |1\rangle_A\left(|0\rangle_B - |1\rangle_B\right)$$

Now, Bob doesn't know if he should measure in the $\{|0\rangle,|1\rangle\}$ or $\{|+\rangle,|-\rangle\}$ basis. Even if he picks the right one, he still doesn't know if the result is right.

➔ Need for communication

EXAMPLE 2.2

$$|\phi_1\rangle = |0\rangle_A |0\rangle_B$$

$$|\phi_2\rangle = |1\rangle_A |+\rangle_B = |1\rangle_A \left(|0\rangle_B + |1\rangle_B\right)$$

$$|\phi_3\rangle = |0\rangle_A |1\rangle_B$$

$$|\phi_4\rangle = |1\rangle_A |-\rangle_B = |1\rangle_A \left(|0\rangle_B - |1\rangle_B\right)$$

Now, what about if Bob measures first? If he measures in the $\{|0\rangle, |1\rangle\}$ basis, he only eliminates one state. Same thing for $\{|+\rangle, |-\rangle\}$.

➔ Need for communication.

MULTI-PARTITE – W/ COMM.

SETTING

A state $|\phi_i\rangle$ from a set $\{|\phi_i\rangle\}$ is seperated into two parts. Alice and Bob are each given one part (which can be entangled together).

MULTI-PARTITE – W/ COMM.

$$|\phi_1\rangle, |\phi_2\rangle, \ldots |\phi_n\rangle ?$$

PROBLEM

Can Alice and Bob can they gain the knowledge* of which one it is using only local operations and classical communication?

MULTI-PARTITE – W/ COMM.

LESS RESTRICTIONS

- They have only one copy
- They can only perform local operations
- The two parties **can** communicate but only **classically** (they can't exchange qubits)

EXAMPLE 2 (revisited)

Now, If Alice and Bob call each other to schedule their measurements and Alice tells her measurement result to Bob, they can distinguish the states perfectly.

$$|\phi_1\rangle = |0\rangle_A |0\rangle_B$$

$$|\phi_2\rangle = |1\rangle_A |+\rangle_B = |1\rangle_A \left(|0\rangle_B + |1\rangle_B\right)$$

$$|\phi_3\rangle = |0\rangle_A |1\rangle_B$$

$$|\phi_4\rangle = |1\rangle_A |-\rangle_B = |1\rangle_A \left(|0\rangle_B - |1\rangle_B\right)$$

EXAMPLE 3

Bell states:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B\right)$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B\right)$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B\right)$$

EXAMPLE 3

1st TRY

$$\left|\phi^{+}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A \left|0\right\rangle_B + \left|1\right\rangle_A \left|1\right\rangle_B\right)$$

$$\left|\phi^{-}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A \left|0\right\rangle_B - \left|1\right\rangle_A \left|1\right\rangle_B\right)$$

$$\left|\psi^{+}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A \left|1\right\rangle_B + \left|1\right\rangle_A \left|0\right\rangle_B\right)$$

$$\left|\psi^{-}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A \left|1\right\rangle_B - \left|1\right\rangle_A \left|0\right\rangle_B\right)$$

Alice measures in the $\{|0\rangle, |1\rangle\}$ basis and gets 0, that doesn't help them. No state is eliminated.

EXAMPLE 3

00 → $|\phi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$

01 → $|\phi^-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B - |1\rangle_A|1\rangle_B\right)$

?

10 → $|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B\right)$

11 → $|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B\right)$

Bob measures in the same basis and gets 1, they still don't know which of two states it was. The second bit is hidden.

EXAMPLE 3

2nd TRY

$$\left|\phi^{+}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A\left|0\right\rangle_B + \left|1\right\rangle_A\left|1\right\rangle_B\right)$$

$$\left|\phi^{-}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A\left|0\right\rangle_B - \left|1\right\rangle_A\left|1\right\rangle_B\right)$$

$$\left|\psi^{+}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A\left|1\right\rangle_B + \left|1\right\rangle_A\left|0\right\rangle_B\right)$$

$$\left|\psi^{-}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A\left|1\right\rangle_B - \left|1\right\rangle_A\left|0\right\rangle_B\right)$$

Bob measures in the $\{\left|+\right\rangle, \left|-\right\rangle\}$ basis and gets -, that eliminates 2 states

EXAMPLE 3

2nd TRY

00 → ~~$\left|\phi^{+}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A\left|0\right\rangle_B + \left|1\right\rangle_A\left|1\right\rangle_B\right)$~~

?

01 → $\left|\phi^{-}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A\left|0\right\rangle_B - \left|1\right\rangle_A\left|1\right\rangle_B\right)$

10 → ~~$\left|\psi^{+}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A\left|1\right\rangle_B + \left|1\right\rangle_A\left|0\right\rangle_B\right)$~~

11 → $\left|\psi^{-}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A\left|1\right\rangle_B - \left|1\right\rangle_A\left|0\right\rangle_B\right)$

If Alice measures in the $\{\left|+\right\rangle, \left|-\right\rangle\}$ basis then she has to get **-**. They still don't know if it was $\left|\psi^{-}\right\rangle$ or $\left|\phi^{-}\right\rangle$.

INDISTINGUISHABILITY

- The four Bell states cannot be distinguished using LOCC.
- One of the two classical bits of information will never be available to Alice and Bob.
- This is called data hiding.

UNEXTENDIBLE PRODUCT BASES

- An Unextendible Product Basis (UPB) is a basis of product states to which we cannot add any other orthogonal state lying inside our space of interest.
- An UPB is indistinguishable using only LOCC.

UPB EXAMPLE

5-dimensional subspace of $\mathcal{H}_9 = \mathcal{H}_3 \otimes \mathcal{H}_3$:

$$\left|\phi_1\right\rangle = \frac{1}{\sqrt{2}}\left|0\right\rangle_A \left(\left|0\right\rangle_B - \left|1\right\rangle_B\right)$$

$$\left|\phi_2\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle_A - \left|1\right\rangle_A\right)\left|2\right\rangle_B$$

$$\left|\phi_3\right\rangle = \frac{1}{\sqrt{2}}\left|2\right\rangle_A \left(\left|1\right\rangle_B - \left|2\right\rangle_B\right)$$

$$\left|\phi_4\right\rangle = \frac{1}{\sqrt{2}}\left(\left|1\right\rangle_A - \left|2\right\rangle_A\right)\left|0\right\rangle_B$$

$$\left|\phi_5\right\rangle = \frac{1}{3}\left(\left|0\right\rangle_A + \left|1\right\rangle_A + \left|2\right\rangle_A\right)\left(\left|0\right\rangle_B + \left|1\right\rangle_B + \left|2\right\rangle_B\right)$$

Tiles states

A UPB OF INTEREST

4-dimensional subspace of

$\mathcal{H}_8 = \mathcal{H}_2 \otimes \mathcal{H}_2 \otimes \mathcal{H}_2$:

$$\left|\phi_1\right\rangle = \left|0\right\rangle_A \left|0\right\rangle_B \left|0\right\rangle_C$$

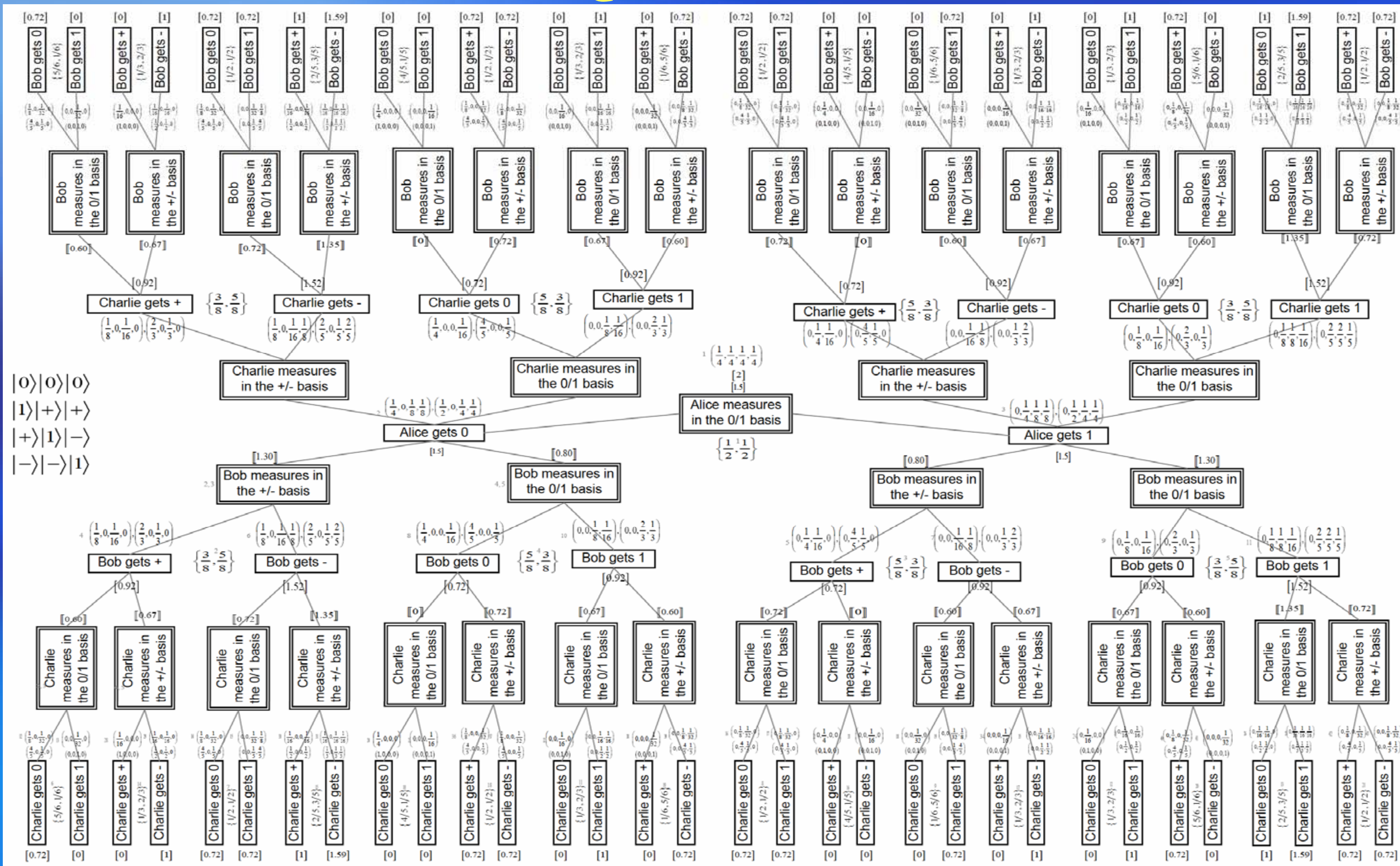
$$\left|\phi_2\right\rangle = \left|1\right\rangle_A \left|+\right\rangle_B \left|+\right\rangle_C$$

$$\left|\phi_3\right\rangle = \left|+\right\rangle_A \left|1\right\rangle_B \left|-\right\rangle_C$$

$$\left|\phi_4\right\rangle = \left|-\right\rangle_A \left|-\right\rangle_B \left|1\right\rangle_C$$

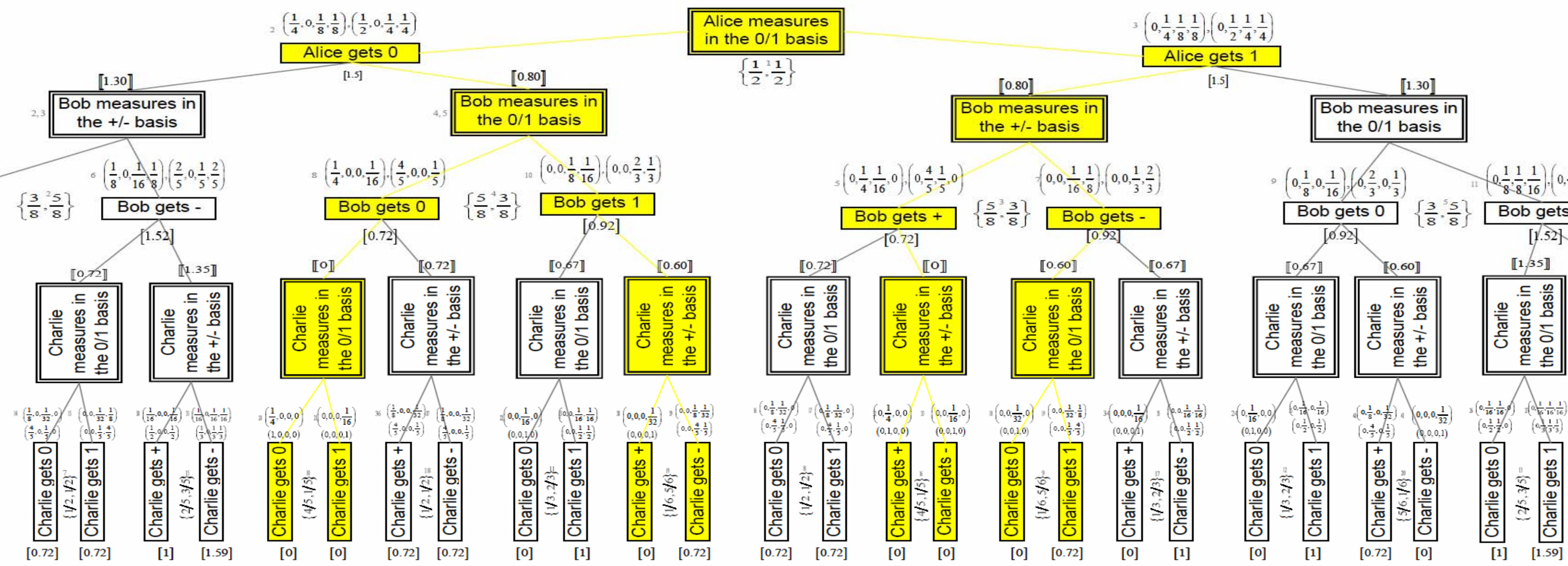
The set of four states is symmetric under permutation of A, B & C (relabeling)

DIFFERENT MEASUREMENT SEQUENCES



OPTIMAL PROTOCOL

Maximum information extraction using $\{|0\rangle,|1\rangle\}$ and $\{|+\rangle,|-\rangle\}$ measurements.



This protocol results in an average extraction of 1.775 bits of information $\left(H_{before} - H_{after}\right)$.

OPTIMAL PROTOCOL

Next step: consider POVMs and see if more information can be extracted.

Work in progress…

OTHER INTERESTING VARIATIONS

- Give multiple copies to each parties
- Communication restrictions between all or some parties
- Permit some quantum communication (transfer qubits)

INTERESTING RESULTS

- 2 orthogonal states distributed between any number of parties can be reliably distinguished (w/ probability 100%)
- Non-locality w/o entanglement
- Some states can’t be distinguished using one copy, but can using multiple copies
- Any state can be distinguished w/ *N-1* copies (for a state from a set of *N* orthogonal states)

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