IQC

Magic States

Presented by Nathan Babcock

Overview

I will discuss the following points:

1. **Quantum Error Correction**
2. **The Stabilizer Formalism**
3. **Clifford Group Quantum Computation**
4. **Magic States**
5. **Derivation of the Distillation Algorithm**
6. **Present & Future Work**

1.1 Quantum Error Correction

The purpose of quantum error correction is to encode qubits redundantly so that they may be recovered in the event of an error.

For example, we might use the encoding

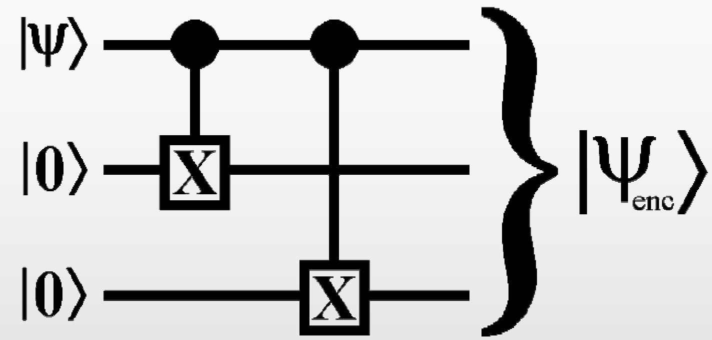
$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$$

This state is resilient against a single bit-flip error (i.e., the Pauli X operator) on any of the three qubits.

1.2 The Three Qubit Code

Consider the encoding

$$|\psi_{enc}\rangle = U(\alpha|0\rangle + \beta|1\rangle)|00\rangle = \alpha|000\rangle + \beta|111\rangle$$



Since a bit-flip error could occur on any qubit, our possible output states after decoding are

No error:	$U^\dagger(\alpha|000\rangle + \beta|111\rangle)$	$=$	$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$
Qubit 1 flip:	$U^\dagger(\alpha|100\rangle + \beta|011\rangle)$	$=$	$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$
Qubit 2 flip:	$U^\dagger(\alpha|010\rangle + \beta|101\rangle)$	$=$	$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$
Qubit 3 flip:	$U^\dagger(\alpha|001\rangle + \beta|110\rangle)$	$=$	$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

Now we only need measure the ancilla qubits in the computational basis $\{|0\rangle, |1\rangle\}$ to determine which error occurred.

Note that this encoding protects against bit-flip errors but not phase-flip errors (the Pauli Z operator). To protect against bit-flip *and* phase-flip errors, more ancillas are necessary!

2.1 The Stabilizer Formalism

The "stabilizer" for a state $|\psi\rangle$ is defined as the group of operators $\{S_i\}$ for which $S_i|\psi\rangle = |\psi\rangle$. For example, the stabilizer for the state $|0\rangle$ is $\{I, Z\}$ since $I|0\rangle = |0\rangle$ and $Z|0\rangle = |0\rangle$.

To begin to see how this formalism is useful to understanding quantum error correction, again consider the encoding:

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle.$$

The state $|\psi\rangle$ is initially stabilized by the group of operators

$$\{III, IIZ, IZI, IZZ\}.$$

The encoded state $|\psi_{enc}\rangle$ is stabilized by the group of operators

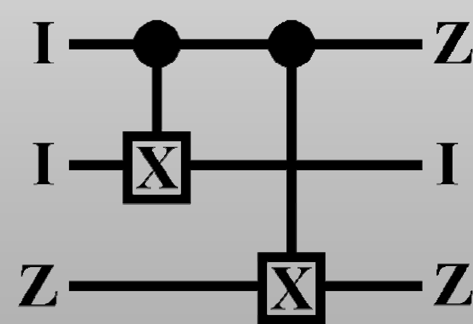
$$\{III, ZIZ, ZZI, IZZ\}.$$

2.2 Stabilizer for the Three Qubit Code

It is straightforward to determine what the stabilizer is for the encoded state $|\psi_{enc}\rangle$, even for more complicated encoding circuits. Just put each operator in the stabilizer group through the circuit and see what comes out!

For the three qubit code, $(\alpha|0\rangle + \beta|1\rangle)|00\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$, the mapping between unencoded and encoded stabilizer elements is

$$\begin{array}{ccc} III & \rightarrow & III \\ IIZ & \rightarrow & ZIZ \\ IZI & \rightarrow & ZZI \\ IZZ & \rightarrow & IZZ \end{array}$$



By choosing circuits with *very specific* encoded stabilizers, we can do quantum error correction.

2.3 How (Non-Degenerate) Codes Work

Step 1: Begin with an arbitrary qubit $|\psi\rangle$ and several ancilla qubits $|00\ldots0\rangle$. This state is stabilized by the identity acting on $|\psi\rangle$ and any permutation of identities and Pauli Z operations acting on the ancillas.

Step 2: Encode to a state with a stabilizer having the property that every one-qubit transformation results in a state with a new, unique stabilizer.

Step 3: Decode the state and measure the ancillas. If all of the ancillas are in the $|0\rangle$ state, we know that either no error occurred or that at least *two* errors occurred (which we cannot fix!). If one error occurred, some of the ancillas will be in the $|1\rangle$ state. Each permutation of $|1\rangle$'s will represent a different output stabilizer, and therefore a different single qubit error which can now be corrected. The permutations of outputted $|1\rangle$'s is called the "error syndrome".

2.4 So How Many Ancillas Already?

Each qubit may suffer one of three errors: a bit-flip error (X), a phase-flip error (Z), or both (Y). Obviously we must also account for the possibility of no errors, so for n qubits there are 3n+1 possible outcomes. Since the n-1 ancillas can denote no more than 2^{n-1} distinguishable states, we require:

$$3n + 1 \leq 2^{n-1}.$$

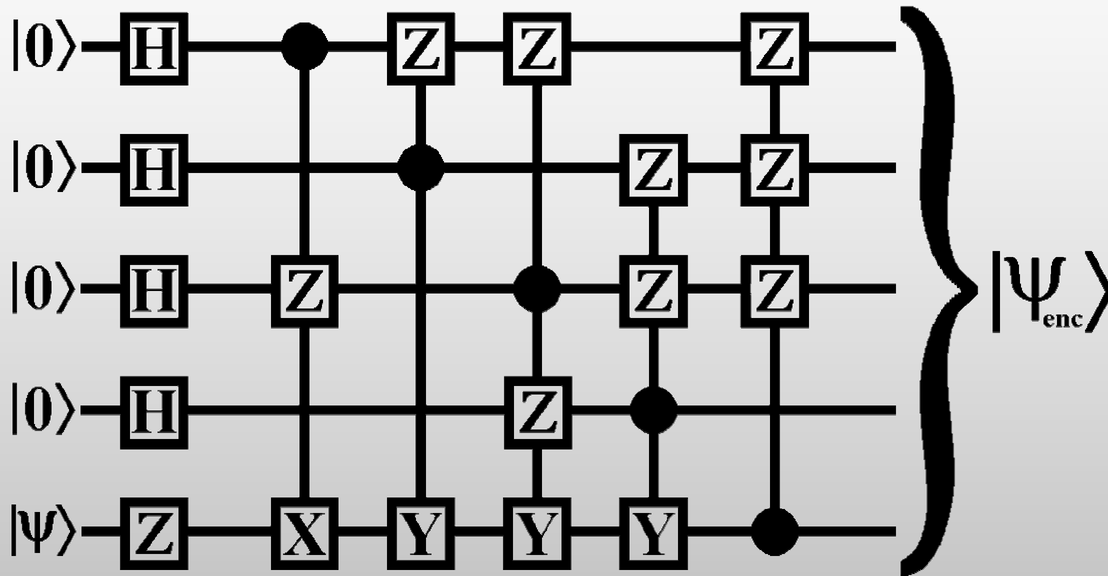
The equality is satisfied when n = 5, so quantum error correction on one qubit requires at least *four* ancilla qubits.

But what, you may wonder, if the error is not merely an X, Y, or Z Pauli operator, but some *superposition* of all three? By measuring a superposition of errors in the {X, Y, Z} basis we collapse it into *just one* of the three possible errors, and the problem is resolved!

2.5 A "Perfect" Five Qubit Code [3]

Input Stabilizer

I I I I I
Z I I I I
I Z I I I
I I Z I I
I I I Z I
Z Z I I I
I Z Z I I
I I Z Z I
Z I Z I I
I Z I Z I
Z I I Z I
Z Z Z I I
Z Z I Z I
Z I Z Z I
I Z Z Z I
Z Z Z Z I



Output Stabilizer

I I I I I
I X Z Z X
X I X Z Z
Z X I X Z
Z Z X I X
X Z Z X I

I Z Y Y Z
Z I Z Y Y
Y Z I Z Y
Y Y Z I Z
Z Y Y Z I

I Y X X Y
Y I Y X X
X Y I Y X
X X Y I Y
Y X X Y I

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

3.1 Clifford Group Quantum Computation

The "Clifford Group" scheme for quantum computation is constructed from the following three primitives:

1. Preparation of qubits in the $|0\rangle$ state.
2. Application of unitary operations from the Clifford Group.
3. Measurement of qubits in the X, Y, and Z bases.

The Clifford Group is the group of operators which maps the Pauli Group {I, X, Y, Z} onto itself. It is defined by the generators

$$\mathbf{C} = \{\mathrm{H}, \mathrm{S}, \mathrm{cnot}, \otimes, \bullet\},$$

where

$$\mathrm{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad \mathrm{S} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}, \quad \mathrm{cnot} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

But this model for quantum computing can be simulated efficiently on a classical computer [1,2], and is therefore not a prototype for universal quantum computation!

3.2 A Modification to Clifford for UQC

The Clifford Group model is a "nice" model, since it has been shown that it is straightforward to implement fault-tolerantly using known error correction algorithms. It is therefore of interest to find a *simple* modification to the Clifford model that can also be implemented fault-tolerantly and will allow universal quantum computation.

It can be shown [4] that an additional primitive such as

4. Preparation of qubits in the state $\cos(\frac{\pi}{8})\left|0\right\rangle + \sin(\frac{\pi}{8})\left|1\right\rangle$

is sufficient to allow quantum computation. (This ancilla is not unique!).

It is therefore of significant interest to find fault-tolerant methods of preparing qubit states such as the one above.

3.3 Bravyi & Kitaev's (Partial) Solution

In quant-ph/0403025, Bravyi & Kitaev demonstrate an ingenious method for purifying such states, granted that they can already be prepared approximately. They examine two states specifically,

$$\left|H\right\rangle = \cos(\tfrac{\pi}{8})\left|0\right\rangle + \sin(\tfrac{\pi}{8})\left|1\right\rangle$$

and

$$|T\rangle = \cos(\beta)|0\rangle + e^{i\pi/4}\sin(\beta)|1\rangle, \quad \beta = \tfrac{1}{2}\arccos(\tfrac{1}{\sqrt{3}})\,.$$

These are the so-called "magic states", since they can be purified using Bravyi & Kitaev's algorithm *and* can be added as ancilla qubits to the Clifford model to allow universal quantum computation.

4.1 Magic States

Here we will examine the so-called "T-type" magic states:

$|T_0\rangle = \cos(\beta)|0\rangle + e^{i\pi/4}\sin(\beta)|1\rangle$ and $|T_1\rangle = \sin(\beta)|0\rangle - e^{i\pi/4}\cos(\beta)|1\rangle$,

$\beta = \frac{1}{2}\arccos(\frac{1}{\sqrt{3}})$.

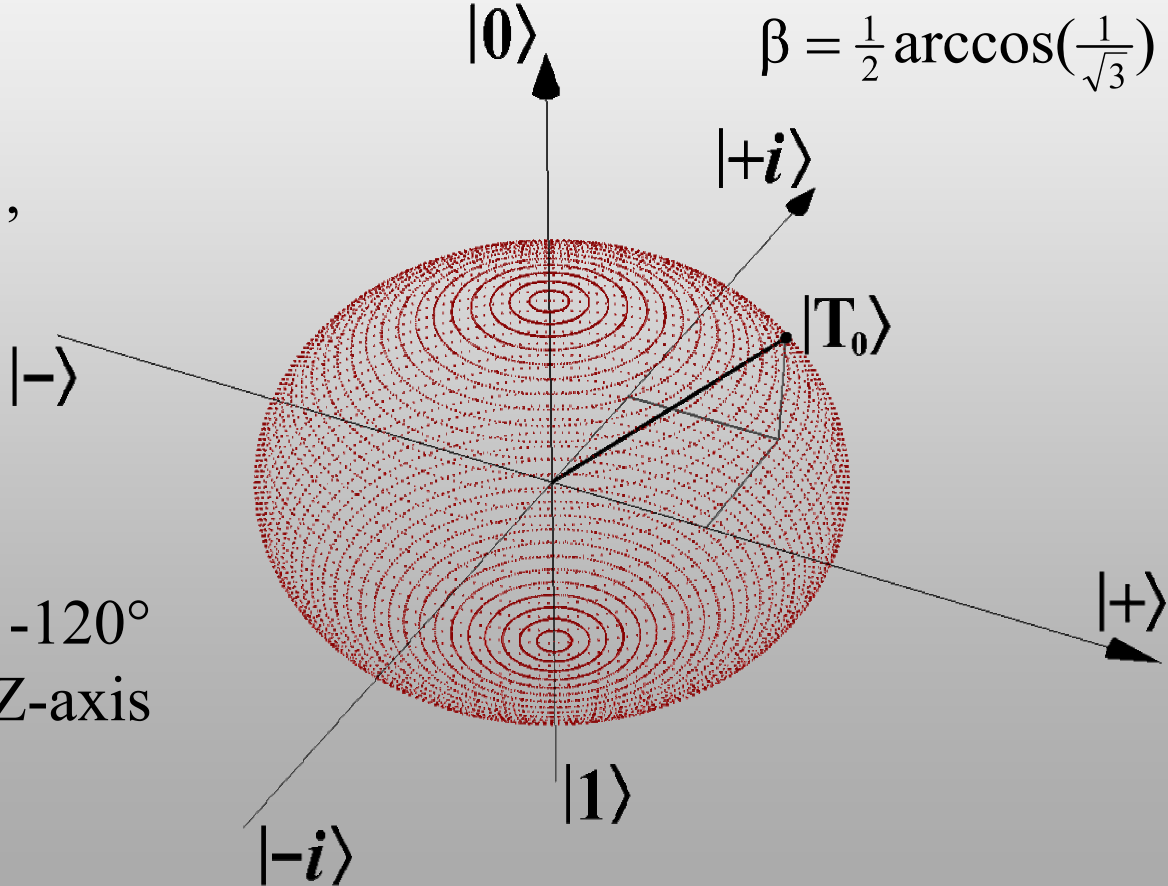
These are eigenstates of

$$T = S \cdot H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix},$$

with eigenvalues

$$T\,|T_0\rangle = e^{i\pi/3}|T_0\rangle,$$
$$T\,|T_1\rangle = e^{-i\pi/3}|T_1\rangle.$$

Geometrically, T is a -120° rotation about the XYZ-axis of the Bloch sphere.



$$|T_0\rangle\langle T_0| = \tfrac{1}{2}[I + \tfrac{1}{\sqrt{3}}(X+Y+Z)]$$

4.2 On the Notation

Consider the pure states $|T_0\rangle$ and $|T_1\rangle$ as density matrices:

$$|T_0\rangle\langle T_0| = \tfrac{1}{2}[I + \tfrac{1}{\sqrt{3}}(X+Y+Z)] \qquad |T_1\rangle\langle T_1| = \tfrac{1}{2}[I - \tfrac{1}{\sqrt{3}}(X+Y+Z)]$$

Suppose, however, that we cannot prepare $|T_0\rangle$ and $|T_1\rangle$ perfectly. Instead, let us prepare partially mixed states that are approximations to $|T_0\rangle$ and $|T_1\rangle$:

$$\rho_{T_0} = \tfrac{1}{2}[I + \tfrac{p}{\sqrt{3}}(X+Y+Z)], \quad \rho_{T_1} = \tfrac{1}{2}[I - \tfrac{p}{\sqrt{3}}(X+Y+Z)].$$

where $0 \le p \le 1$. When $p = 1$, a state is pure. When $p = 0$, it is the completely mixed state. In general, we can write these mixed states as sums of $|T_0\rangle\langle T_0|$ and $|T_1\rangle\langle T_1|$ in the following way:

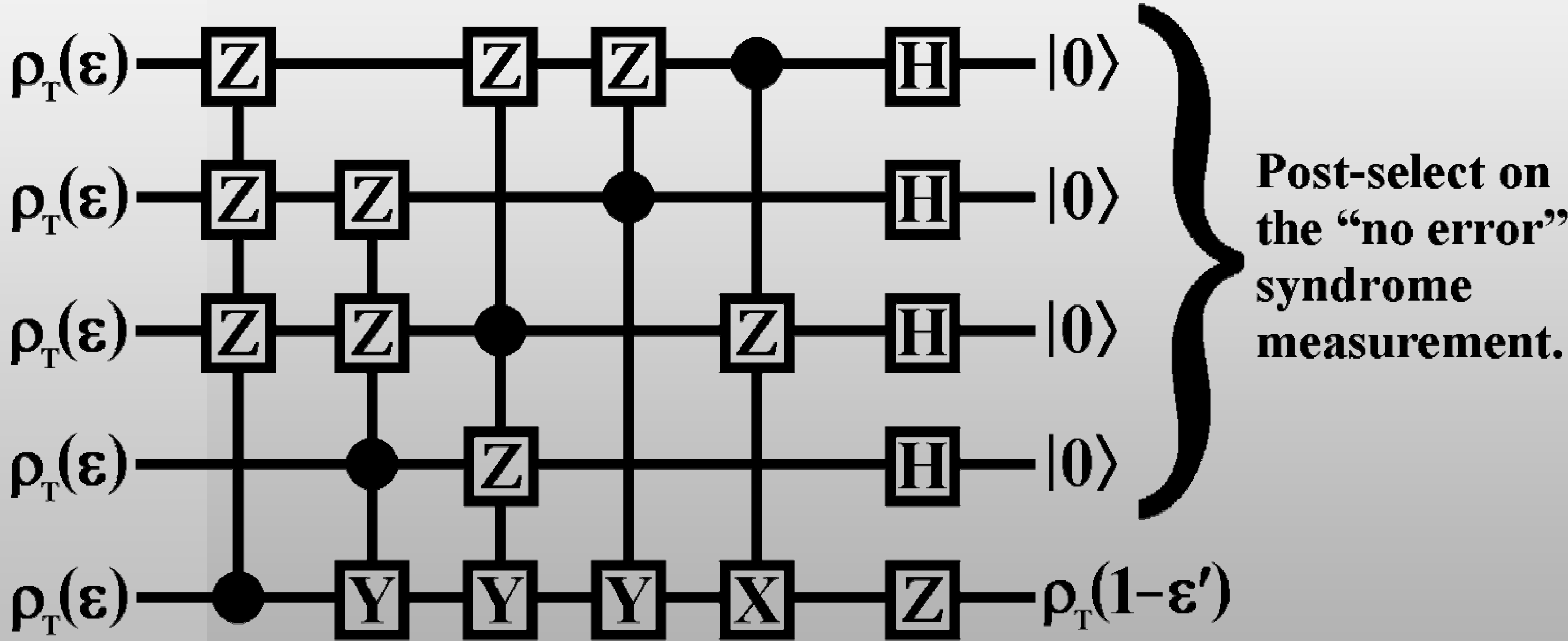
$$\rho_T(\varepsilon) = (1-\varepsilon)|T_0\rangle\langle T_0| + \varepsilon|T_1\rangle\langle T_1|.$$

Using this notation, the completely mixed state is

$$\rho_T(\tfrac{1}{2}) = \tfrac{1}{2}|T_0\rangle\langle T_0| + \tfrac{1}{2}|T_1\rangle\langle T_1| = \tfrac{1}{2}I = \begin{bmatrix} \tfrac{1}{2} & 0 \\ 0 & \tfrac{1}{2} \end{bmatrix}.$$

4.3 The Circuit

The purification circuit takes five approximately prepared “magic states” and probabilistically returns an improved one:



Here, $\varepsilon' < \varepsilon$ when $\varepsilon < \frac{1}{2}\left(1-\sqrt{\frac{3}{7}}\right) \approx 0.173$. $\rho_T(\varepsilon) = (1-\varepsilon)|T_0\rangle\langle T_0| + \varepsilon |T_1\rangle\langle T_1|$

4.4 The Algorithm

The purification algorithm itself quite straightforward:

1. Prepare five identical qubits in the state

$$\rho_T(\varepsilon) = (1-\varepsilon)|T_0\rangle\langle T_0| + \varepsilon|T_1\rangle\langle T_1|$$

2. Apply the decoding operation of the five qubit error correcting code with the stabilizer generated by

$$\{IXZZX, XIXZZ, ZXIXZ, ZZXIX\}.$$

3. Measure the ancilla qubits in the computational basis. If the syndrome $|0000\rangle$ is measured, the output state is

$$\rho_T'(\varepsilon') = \varepsilon'|T_0\rangle\langle T_0| + (1-\varepsilon')|T_1\rangle\langle T_1|$$

where

$$\varepsilon' = \frac{\varepsilon^5 + 5\varepsilon^2(1-\varepsilon)^3}{\varepsilon^5 + 5\varepsilon^2(1-\varepsilon)^3 + 5\varepsilon^3(1-\varepsilon)^2 + (1-\varepsilon)^5}$$

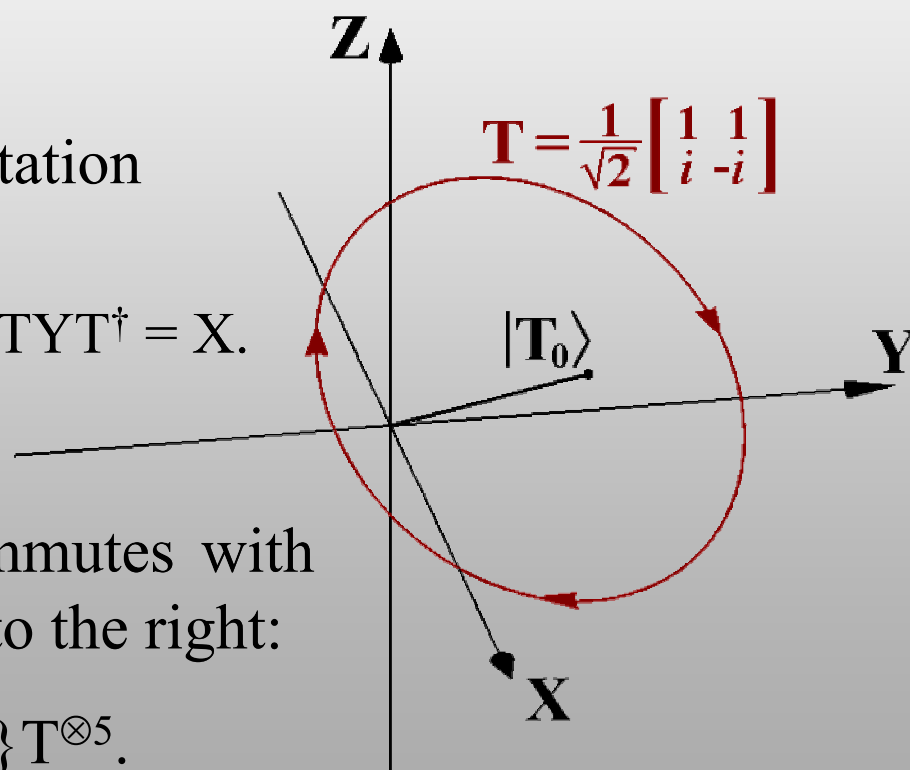
4. Repeat the algorithm recursively until enough qubits of the desired purity have been produced.

5.1 Why Does It Work?

The most important thing to realize is that although T is a member of the Clifford Group, its eigenstates $|T_0\rangle$ and $|T_1\rangle$ can be used as ancillas to make quantum gates that are not!

Since T is a -120° rotation about the XYZ-axis,

$$TXT^\dagger = Z,\ TZT^\dagger = Y,\ TYT^\dagger = X.$$



As a result, $T^{\otimes 5}$ commutes with the stabilizer shown to the right:

$$T^{\otimes 5}\{S_i\} = \{S_i\}T^{\otimes 5}.$$

Encoded Stabilizer

I I I I I

I X Z Z X
X I X Z Z
Z X I X Z
Z Z X I X
X Z Z X I

I Z Y Y Z
Z I Z Y Y
Y Z I Z Y
Y Y Z I Z
Z Y Y Z I

I Y X X Y
Y I Y X X
X Y I Y X
X X Y I Y
Y X X Y I

5.2 T is Transversal

Not only does $T^{\otimes 5}$ commute with the stabilizer, it turns out that $T^{\otimes 5} = T_{enc}$. To see this, examine $|T_{0enc}\rangle$ and $|T_{1enc}\rangle$ closely:

$$\begin{aligned}
\left|T_{0\mathrm{enc}}\right\rangle &= \tfrac{1}{\sqrt{6}}\left|T_1T_1T_1T_1T_1\right\rangle \\
&+\left(\tfrac{1+\sqrt{3}i}{4\sqrt{3}}\right)\left(\left|T_0T_0T_0T_1T_1\right\rangle+\left|T_0T_0T_1T_1T_0\right\rangle+\left|T_0T_1T_1T_0T_0\right\rangle+\left|T_1T_1T_0T_0T_0\right\rangle+\left|T_1T_0T_0T_0T_1\right\rangle\right) \\
&+\left(\tfrac{1-\sqrt{3}i}{4\sqrt{3}}\right)\left(\left|T_0T_0T_1T_0T_1\right\rangle+\left|T_0T_1T_0T_0T_1\right\rangle+\left|T_0T_1T_0T_1T_0\right\rangle+\left|T_1T_0T_0T_1T_0\right\rangle+\left|T_1T_0T_1T_0T_0\right\rangle\right),
\end{aligned}$$

$$\begin{aligned}
\left|T_{1\mathrm{enc}}\right\rangle &= \tfrac{1}{\sqrt{6}}\left|T_0T_0T_0T_0T_0\right\rangle \\
&-\left(\tfrac{1-\sqrt{3}i}{4\sqrt{3}}\right)\left(\left|T_1T_1T_1T_0T_0\right\rangle+\left|T_1T_1T_0T_0T_1\right\rangle+\left|T_1T_0T_0T_1T_1\right\rangle+\left|T_0T_0T_1T_1T_1\right\rangle+\left|T_0T_1T_1T_1T_0\right\rangle\right) \\
&-\left(\tfrac{1+\sqrt{3}i}{4\sqrt{3}}\right)\left(\left|T_1T_1T_0T_1T_0\right\rangle+\left|T_1T_0T_1T_1T_0\right\rangle+\left|T_1T_0T_1T_0T_1\right\rangle+\left|T_0T_1T_1T_0T_1\right\rangle+\left|T_0T_1T_0T_1T_1\right\rangle\right).
\end{aligned}$$

It is clear that $T^{\otimes 5}|T_{0enc}\rangle = e^{i\pi/3}|T_{0enc}\rangle$ and $T^{\otimes 5}|T_{1enc}\rangle = e^{-i\pi/3}|T_{1enc}\rangle$, and since $T^{\otimes 5}$ commutes with the stabilizer (i.e., it does not bring states into or out of the encoded subspace), we know that $T_{enc} = T^{\otimes 5}$.
(Note, however, that T_{enc} is not unique!)

5.3 Projecting onto the Stabilizer

Now, we see that the mixed state $\rho_T(\varepsilon)^{\otimes 5} = [(1-\varepsilon)|T_0\rangle\langle T_0| + \varepsilon|T_1\rangle\langle T_1|]^{\otimes 5}$ is closely related to the encoded states $|T_{0enc}\rangle$ and $|T_{1enc}\rangle$. When we decode and measure the ancilla qubits, if we obtain the "no error" syndrome $|0000\rangle$ then we know we have projected the state onto the (decoded) stabilizer subspace.

Of course, it makes no difference whether we project onto the stabilizer subspace before or after we decode, since decoding is just a basis transformation from one stabilizer subspace to another.

So, let's consider the effect of projecting the state $\rho_T(\varepsilon)^{\otimes 5}$ directly onto the stabilizer subspace without decoding first. Our stabilizer projector, Π, can be defined in a few equivalent ways:

$$\Pi = \left|0_{enc}\right\rangle\left\langle 0_{enc}\right| + \left|1_{enc}\right\rangle\left\langle 1_{enc}\right| = \left|T_{0_{enc}}\right\rangle\left\langle T_{0_{enc}}\right| + \left|T_{1_{enc}}\right\rangle\left\langle T_{1_{enc}}\right|$$

$$= \tfrac{1}{16}(I^{\otimes 5} + IXZZX)(I^{\otimes 5} + XIXZZ)(I^{\otimes 5} + ZXIXZ)(I^{\otimes 5} + ZZXIX)$$

5.4 The Effect of Π on $\rho_T(\varepsilon)^{\otimes 5}$

Recall that $\Pi = |T_{0enc}\rangle\langle T_{0enc}| + |T_{1enc}\rangle\langle T_{1enc}|$ and that

$$\left|T_{0\,enc}\right\rangle = \tfrac{1}{\sqrt{6}}\left|T_{11111}\right\rangle + \left(\tfrac{1+\sqrt{3}i}{4\sqrt{3}}\right)\left(\left|T_{00011}\right\rangle + \ldots + \left|T_{11000}\right\rangle\right) + \left(\tfrac{1-\sqrt{3}i}{4\sqrt{3}}\right)\left(\left|T_{00101}\right\rangle + \ldots + \left|T_{10100}\right\rangle\right),$$

$$\left|T_{1\,enc}\right\rangle = \tfrac{1}{\sqrt{6}}\left|T_{00000}\right\rangle - \left(\tfrac{1-\sqrt{3}i}{4\sqrt{3}}\right)\left(\left|T_{11100}\right\rangle + \ldots + \left|T_{00111}\right\rangle\right) - \left(\tfrac{1+\sqrt{3}i}{4\sqrt{3}}\right)\left(\left|T_{11010}\right\rangle + \ldots + \left|T_{01011}\right\rangle\right).$$

Our initial state is $\rho_T(\varepsilon)^{\otimes 5} = [(1\text{-}\varepsilon)|T_0\rangle\langle T_0| + \varepsilon|T_1\rangle\langle T_1|]^{\otimes 5}$:

$$\begin{aligned}\rho_T(\varepsilon)^{\otimes 5} = {} & (1\text{-}\varepsilon)^5|T_{00000}\rangle\langle T_{00000}| + \varepsilon(1\text{-}\varepsilon)^4(\,|T_{00001}\rangle\langle T_{00001}| + \ldots\,) \\ & + \varepsilon^2(1\text{-}\varepsilon)^3(\,|T_{00011}\rangle\langle T_{00011}| + \ldots\,) + \varepsilon^3(1\text{-}\varepsilon)^2(\,|T_{00111}\rangle\langle T_{00111}| + \ldots\,) \\ & + \varepsilon^4(1\text{-}\varepsilon)\,(\,|T_{01111}\rangle\langle T_{01111}| + \ldots\,) + \varepsilon^5|T_{11111}\rangle\langle T_{11111}|\end{aligned}$$

Since Π contains no terms like $|T_{00001}\rangle\langle T_{00001}|$ or $|T_{01111}\rangle\langle T_{01111}|$, these terms in $\rho_T(\varepsilon)^{\otimes 5}$ will be annihilated.

Terms like $|T_{00011}\rangle\langle T_{00011}|$ and $|T_{11111}\rangle\langle T_{11111}|$ will project to $|T_{0enc}\rangle\langle T_{0enc}|$.

Terms like $|T_{00111}\rangle\langle T_{00111}|$ and $|T_{00000}\rangle\langle T_{00000}|$ will project to $|T_{1enc}\rangle\langle T_{1enc}|$.

5.5 The New Coefficients

We showed that the projector $\Pi = |T_{0enc}\rangle\langle T_{0enc}| + |T_{1enc}\rangle\langle T_{1enc}|$ maps every term in $\rho_T(\varepsilon)^{\otimes 5}$ to either $|T_{0enc}\rangle\langle T_{0enc}|$ or $|T_{1enc}\rangle\langle T_{1enc}|$. Thus,

$$\rho_T{}'_{enc}(\varepsilon') = N(\varepsilon)\Pi\rho_T(\varepsilon)^{\otimes 5}\Pi = \varepsilon'|T_{0enc}\rangle\langle T_{0enc}| + (1\text{-}\varepsilon')|T_{1enc}\rangle\langle T_{1enc}|$$

where $N(\varepsilon)$ is the appropriate normalization after projecting.

All that remains to be done is to find the value of ε'. It is given by the square of the "fidelity" of $\rho_T(\varepsilon)^{\otimes 5}$ with $|T_{0enc}\rangle$ and $|T_{1enc}\rangle$:

$$\frac{\varepsilon'}{N} = \left\langle T_{0_{enc}} \middle| \rho(\varepsilon)^{\otimes 5} \middle| T_{0_{enc}} \right\rangle = \frac{\varepsilon^5 + 5\varepsilon^2(1-\varepsilon)^3}{6},$$

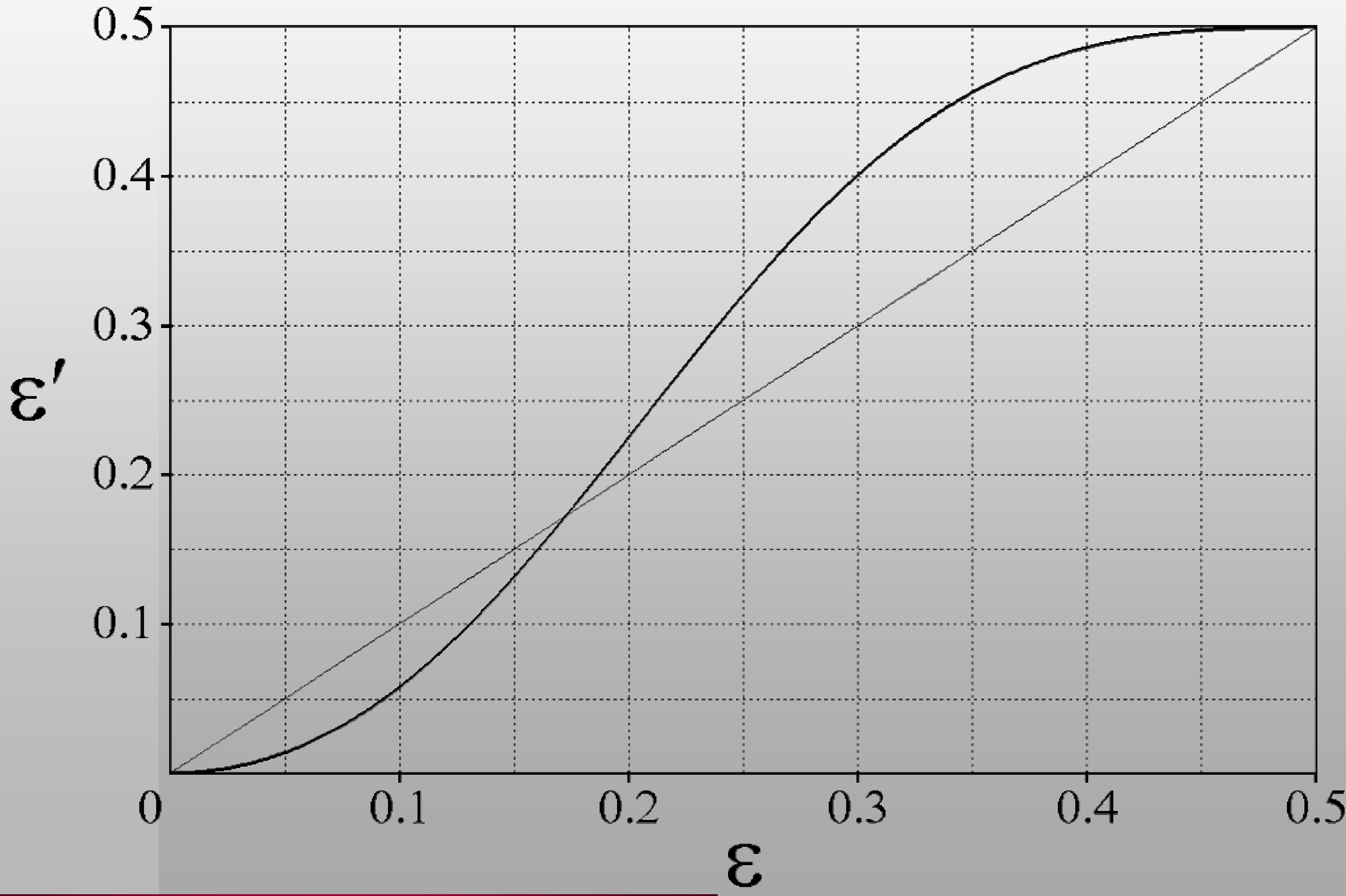
$$\frac{1-\varepsilon'}{N} = \left\langle T_{1_{enc}} \middle| \rho(\varepsilon)^{\otimes 5} \middle| T_{1_{enc}} \right\rangle = \frac{(1-\varepsilon)^5 + 5\varepsilon^3(1-\varepsilon)^2}{6}.$$

Thus,

$$\varepsilon' = \frac{\varepsilon^5 + 5\varepsilon^2(1-\varepsilon)^3}{\varepsilon^5 + 5\varepsilon^2(1-\varepsilon)^3 + 5\varepsilon^3(1-\varepsilon)^2 + (1-\varepsilon)^5}$$

5.6 Graph of ε′ vs. ε

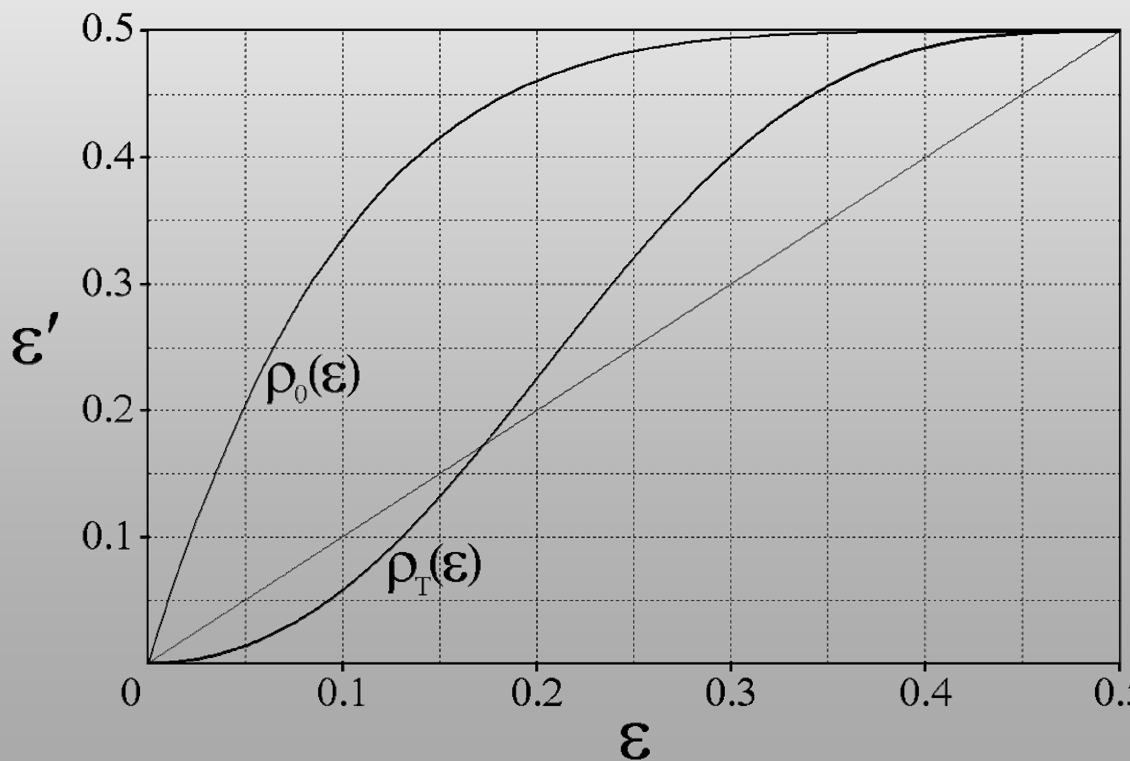
Of course, purification will only occur for values of ε for which ε′ (recursively) converges to either 0 or 1.



5.7 Magic!

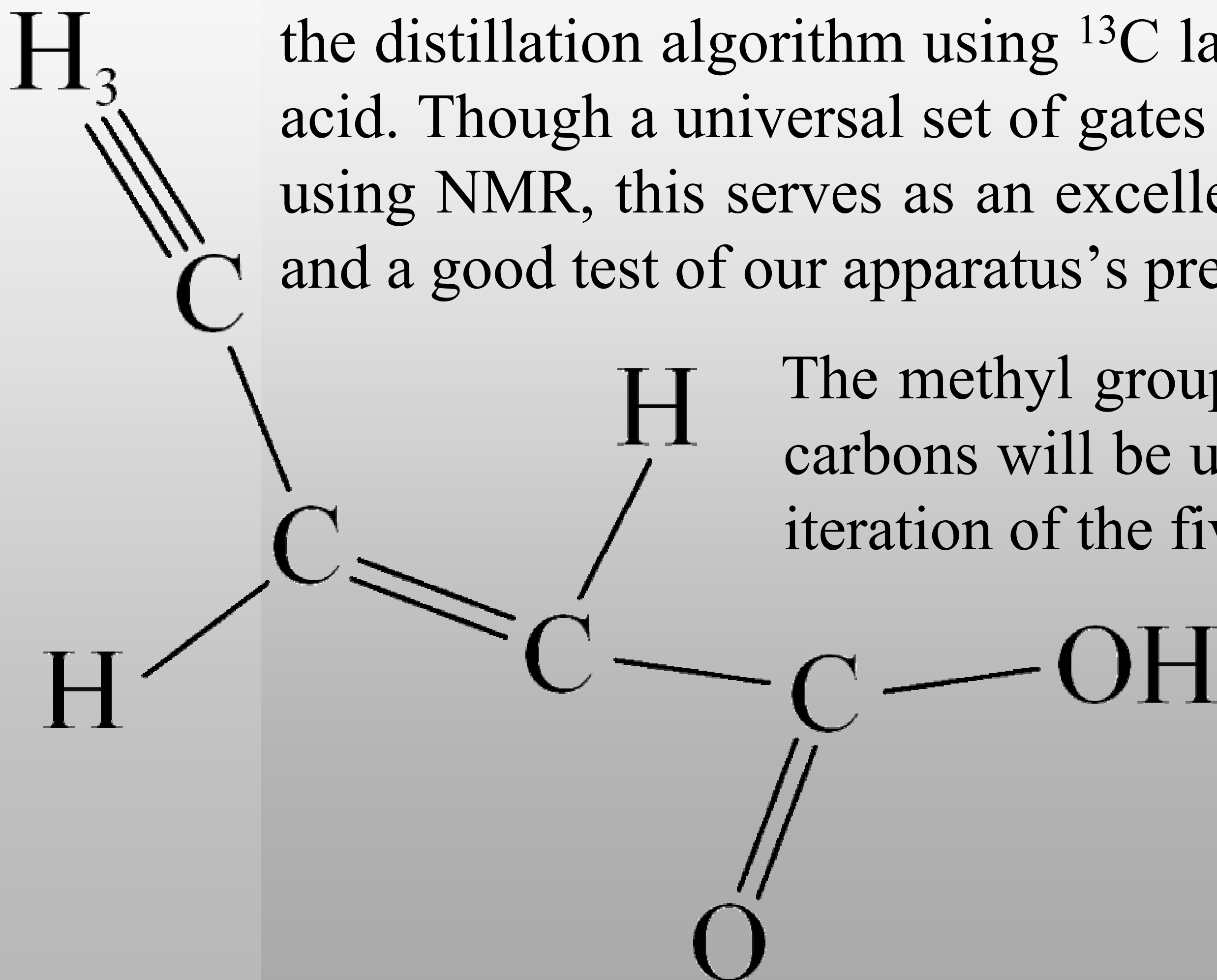
Do *not* think that this procedure will purify the eigenstate of just any operator with the property $U_{enc} = U^{\otimes 5}$. When we project, the coefficients of the old density matrix must be rearranged *just right* for there to be an improvement to the new ones.

The states $|0\rangle$ and $|1\rangle$ are eigenstates of Z, and though $Z_{enc} = Z^{\otimes 5}$ for this stabilizer, the state $\rho_0(\varepsilon) = (1-\varepsilon)|0\rangle\langle 0| + \varepsilon|1\rangle\langle 1|$ will not purify using this method, as shown in the graph.



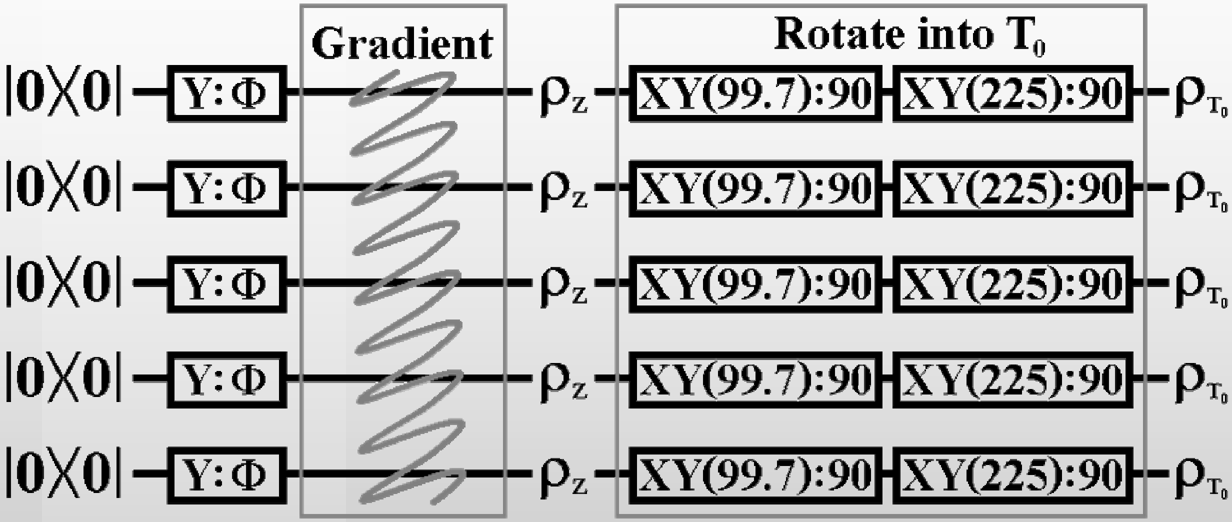
6.1 Implementation

We are currently developing an NMR implementation of the distillation algorithm using ^{13}C labeled trans-crotonic acid. Though a universal set of gates can already be done using NMR, this serves as an excellent proof-of-concept and a good test of our apparatus's precision.

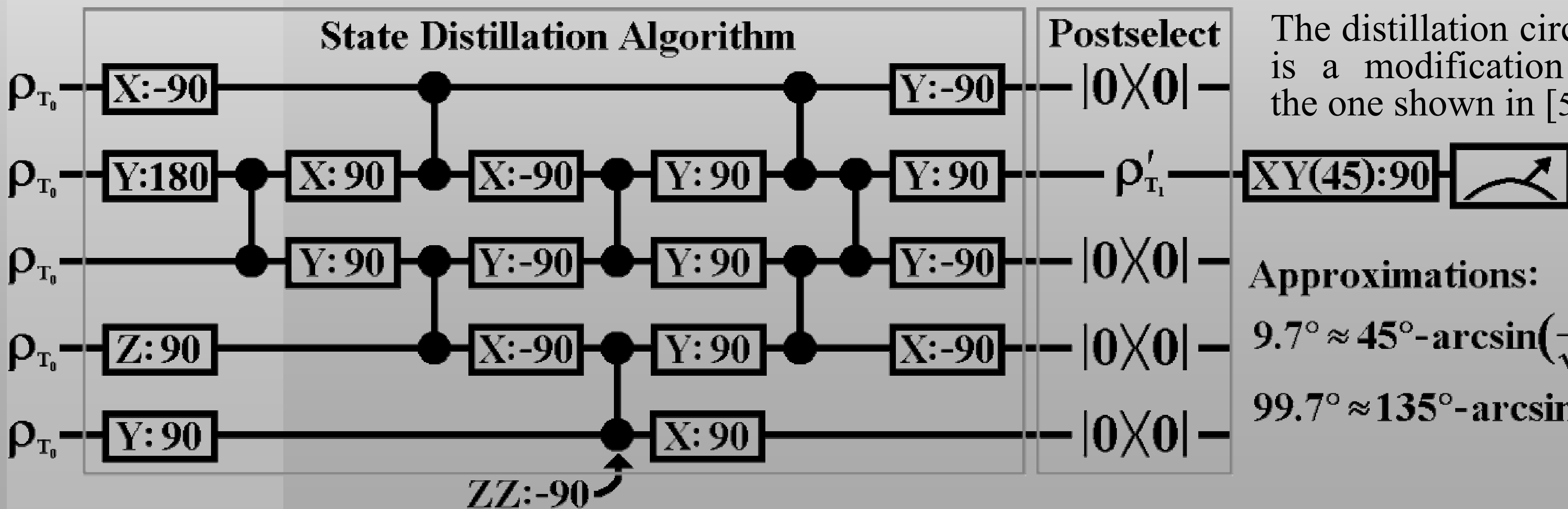


The methyl group (H_3) and the four carbons will be used to perform one iteration of the five-qubit algorithm.

6.2 The NMR Circuit



The mixed state is prepared along the Z-axis, and then rotated into the T-basis. The distillation algorithm is performed, and the output on the post-selected, purified pseudo-qubit is measured.



The distillation circuit is a modification to the one shown in [5].

Approximations:

$9.7° \approx 45°\text{-arcsin}(\frac{1}{\sqrt{3}})$

$99.7° \approx 135°\text{-arcsin}(\frac{1}{\sqrt{3}})$

6.3 Future Work

While it remains uncertain whether Bravyi & Kitaev’s model can be of use for devising a practical quantum computer, it provides a fascinating example of the emergent properties of multi-qubit systems. We are left with a number of interesting questions:

1. Is there a compact way to characterize the states that can be purified using a given stabilizer?
2. Could such an algorithm have any other useful properties (e.g., state purification when measurements are not ideal)?
3. What other surprising uses might error correcting circuits have, in general?

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